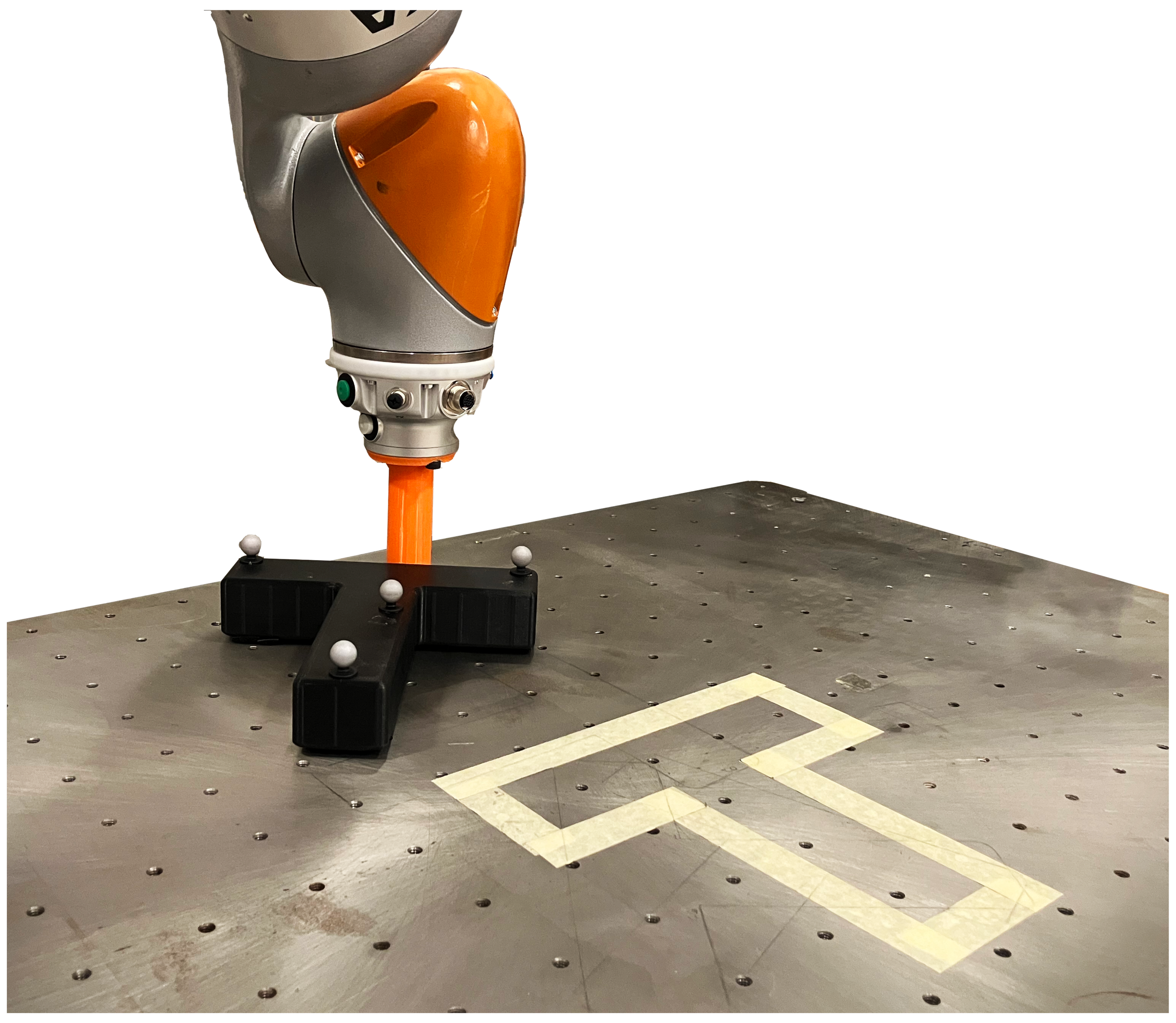

Towards Tight Convex Relaxations for Contact-Rich Manipulation

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Can we generate high-quality data for contact-rich manipulation with trajectory optimization?

INTRODUCTION

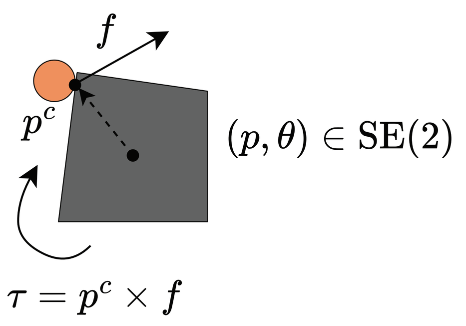
Trajectory optimization for contact-rich manipulation tasks is hard even for simple problems. Consider non-prehensile manipulation in the plane (pictured above):

1. **Hybrid system:**
   - Discrete: where to make contact?
   - Continuous: what continuous motion in each mode?
2. **Underactuated system:**
   - Object can only move when pushed, and force must lie within friction cone.
3. **Collision-free planning** (when contact is not intended)

HIGH-LEVEL APPROACH

1. Approximate planning in a fixed contact mode (nonconvex) with a convex Semidefinite Programming (SDP) relaxation.
2. Build a Graph-of-Convex-Sets (GCS), where each vertex corresponds to planning within a fixed contact mode.
3. Approximately solve the Shortest-Path-Problem (SPP) in the GCS as a (convex) SDP, and retrieve a feasible solution with a quick rounding step.

Step 1: Convex relaxation of nonconvex contact dynamics



Quasi-static contact dynamics are nonconvex when simultaneously optimizing over poses, contact locations and contact forces.

Quadratically Constrained Quadratic Program (Nonconvex)

$$\begin{aligned} \text{minimize} \quad & x^\top Q_0 x \\ \text{subject to} \quad & x^\top Q_i x \geq 0, \\ & Ax \geq 0 \end{aligned}$$

Relaxation →

← *Exact when $X = xx^\top$*

Semidefinite Relaxation (Convex)

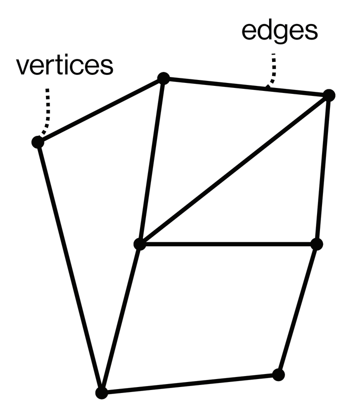
$$\begin{aligned} \text{minimize} \quad & \operatorname{tr}(Q_0 X) \\ \text{subject to} \quad & \operatorname{tr}(Q_i X) \geq 0, \\ & AXe_1 \geq 0, \\ & AXA^\top \geq 0, \\ & X = \begin{bmatrix} 1 & y^\top \\ y & Y \end{bmatrix} \succeq 0 \end{aligned}$$

We formulate the nonconvex planning problem as a QCQP, which we relax into a first-order SDP relaxation, adding tightening constraints to improve the relaxation.

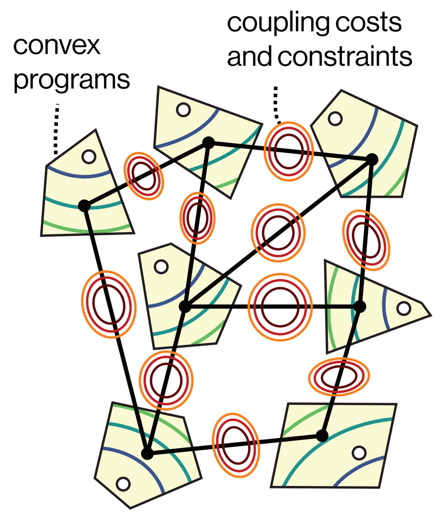
The result is a tight convex approximation of the contact dynamics.

Step 2: Planning across contact modes

We build a Graph of Convex Sets (GCS), with one vertex per contact mode.



A vertex = Semidefinite relaxation of motion planning in a (fixed) contact mode



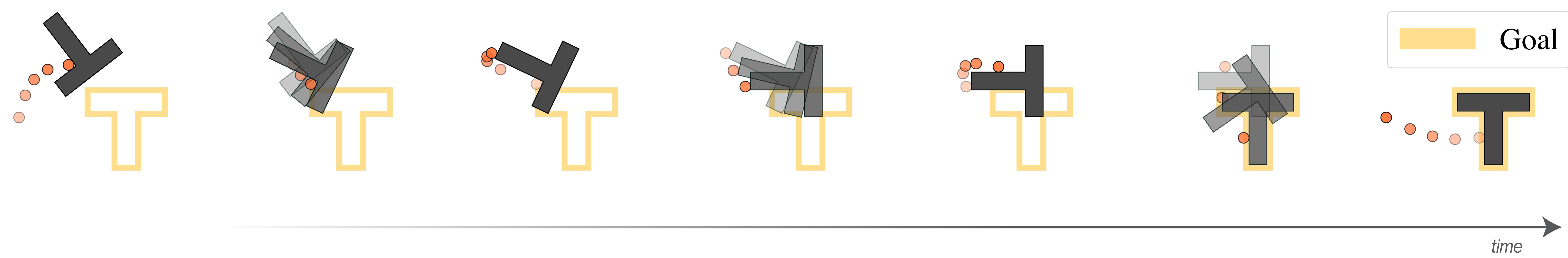
Step 3: Solving the SPP in the GCS

Finally, we solve the Shortest Path Problem (SPP) in the GCS.

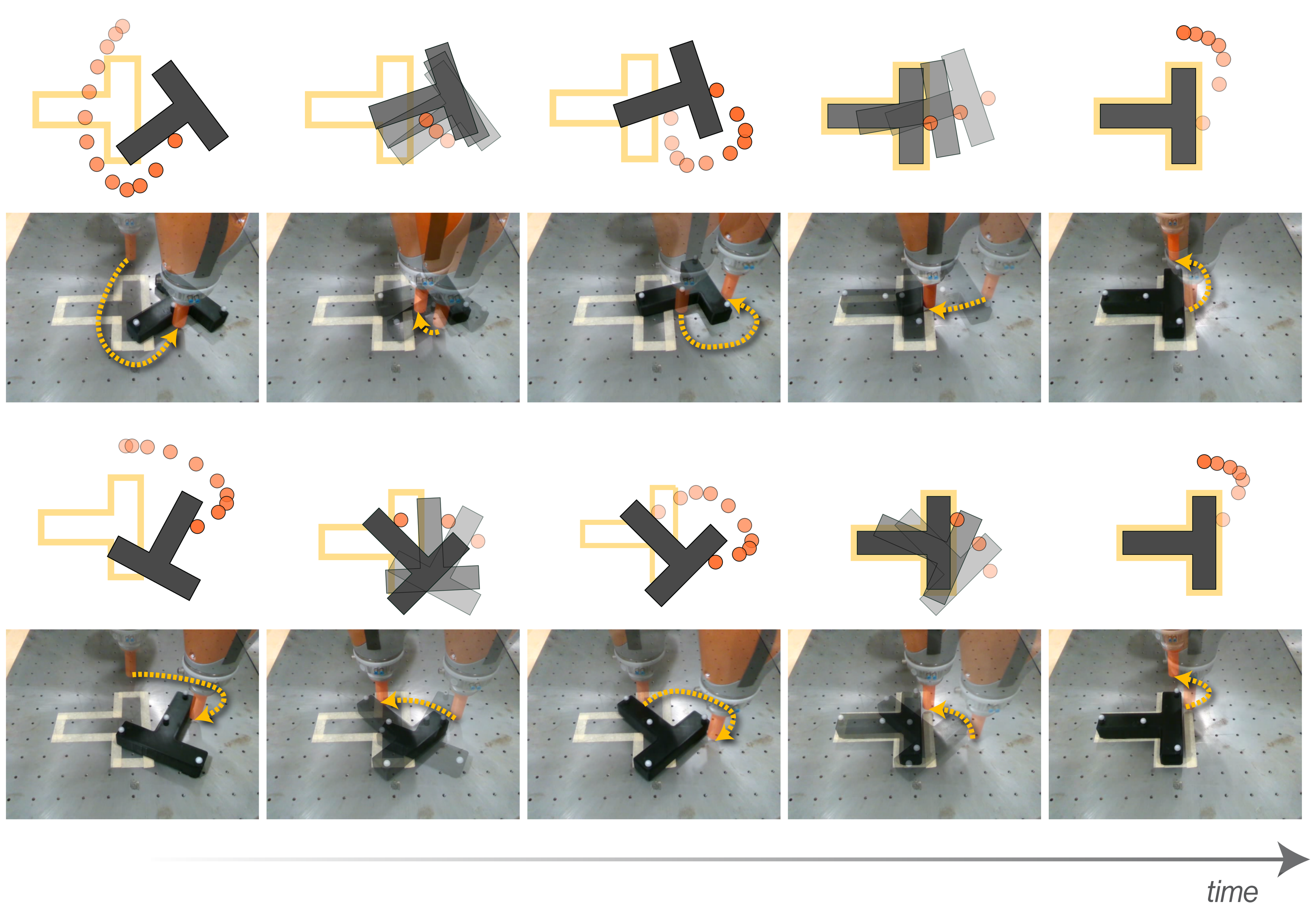
This is a Mixed-Integer Convex Program (MICP) with a strong convex relaxation.

We solve the relaxed problem (a convex SDP) and (quicky) round to a feasible solution.

Example trajectory



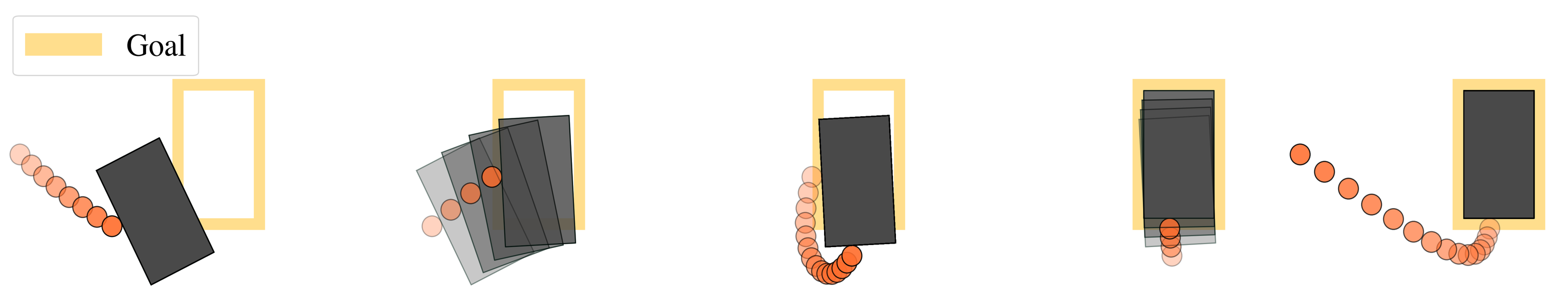
Hardware demonstrations



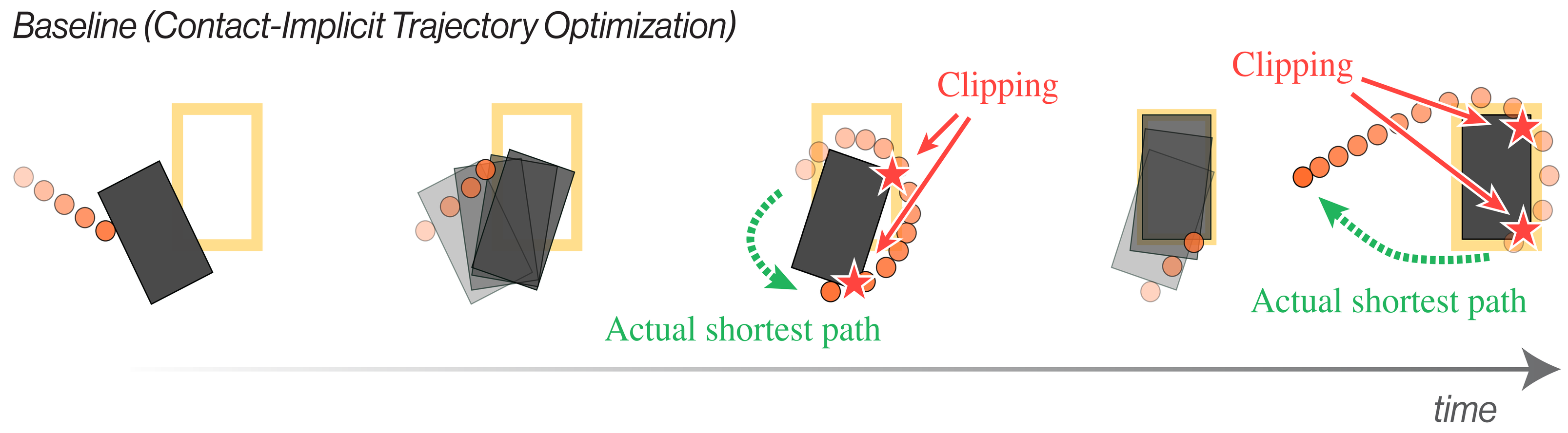
Baseline comparison

Slider	Success Rate: Our method	Success Rate: Contact-implicit method
Box	✓ 100%	✗ 58%
Tee	100%	12%

Our method:



Baseline (Contact-Implicit Trajectory Optimization)



Solve times and optimality gap

Slider	SDP solve time	Rounding time	Optimality gap
Box	7.05s (6.87s)	0.05s (0.05s)	8.33% (5.39%)
Tee	83.61s (80.12s)	0.36s (0.014s)	10.41% (7.47%)

Numbers are mean values, with median values in parentheses.

KEY PROPERTIES

1. *Global:* A few percent from global optimality.
2. *Reliable:* 100% success rate (on tested problem instances).
3. *Efficient:* Scales polynomially (in planning horizon and object geometry complexity).

READ THE PAPER

arXiv
arxiv.org/abs/2402.10312

Project webpage
bernhardgraesdal.com/rss24-towards-tight-convex-relaxations